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Autonomous Vehicle Longitudinal Following Control Based On Model Predictive Control

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Abstract: In this paper, an approach to control the longitudinal motion as well as reject disturbance of a platoon of autonomous vehicles is presented. Firstly, we consider the longitudinal acceleration and velocity of the leading vehicle as disturbance, and establish a model which describes the longitudinal dynamics of inter-vehicle. Secondly, constant time headway (CTH) strategy is adopted to design a longitudinal upper controller based on model predictive control (MPC) to obtain the desired longitudinal acceleration. For rejecting unknown disturbance, a major contribution that differs this paper from previous research is that the upper level controller has two components: a nominal control action and an ancillary control law. The lower level controller adopts simple PID algorithm to determine either a throttle or brake input. Finally, simulations are carried out using vehicle dynamics software veDYNA to verify the effectiveness of the proposed method. Simulation results show that the controller is able to follow the desired spacing as well as reduce the fluctuations of the leading vehicle.

Key Words: Reject Disturbance, Longitudinal Following, Model Predictive Control(MPC)

1 Introduction

Motion control of a platoon of autonomous vehicles has become one of the central issues of current intelligent traffic due to the emerging problems of traffic congestion. It is defined by a list of vehicles driving along the same path together with a close spacing on the premise of safety[1]. Motion control of a platoon of autonomous vehicles can not only specify vehicle driving, gain road capacity but also effectively improve traffic flow. This study includes the lateral and longitudinal automatic control technology, road-vehicle information interactive technology. This paper only studies the longitudinal following control.

The longitudinal control system can be divided into two categories according to different implementations: direct and hierarchical[2]. There exist many control methods based on direct longitudinal control[3, 4]. The longitudinal dynamic model of a platoon of vehicles is a nonlinear system with complex multivariable, and it is affected by dynamic target and obstacles in front. Hence, it is difficult to meet so many performances by designing only one controller[5]. Some scholars divided the control system as two layers inspired by the concept of hierarchical modular. For the hierarchical longitudinal control system, several control methods can be found in literatures. The University of Tokyo[6] designed a feedforward-plus-feedback lower level controller based on the H-∞ robust control method, it's robustness and stability was confirmed by experimentals. In 2006, Gao Feng et.al[7, 8] in Tsinghua University designed a multi-model hierarchical switching control system for vehicle longitudinal acceleration/deceleration based on robust control theory. Zhang Lei[9] proposed an upper control strategy of vehicle longitudinal driving assistance system and an online self-learning method of driver characteristics based on the analysis of driver car-following experimental data as well as aberrant behavior questionnaire, it was also applied to the driving assistance system.

Overviewing the longitudinal control of autonomous vehicle, it is not difficult to find that hierarchical control may be more suitable for this control problem due to a clear division of labour between the upper, lower level controller. However, almost everyone focuses on a single vehicle consisting of an engine, a torque converter, an automatic transmission etc while modeling, but doesn't consider the longitudinal dynamics model of inter-vehicle, not to mention the impact of the leading vehicle acceleration and velocity, therefore, this part is the focus of this study. In this paper, we consider the problem of establishing the longitudinal dynamics model of inter-vehicle, as well as the impact of the acceleration and velocity of the leading vehicle . The longitudinal following of vehicle is regarded as a multi-objective optimization problem and it will be solved under the framework of model predictive control(MPC). MPC is a promising candidate for controlling systems[10, 11]. It exploits a model of the system dynamics to predict the future system evolution and select the first element of the optimal solutions as the control law at current time.

The rest of the paper is structured as follows. The architecture of the longitudinal spacing control system is described in Section 2. The longitudinal dynamics model of inter-vehicle is given in Section 3. Section 4 designs a controller based on MPC which consists of a nominal control action and an ancillary control law. In Section 5, simulation results and discussions are provided. Section 6 is the conclusion of this paper.

2 Architecture of Longitudinal Control System

The main goal of the proposed control law is to achieve the longitudinal following control of a platoon of vehicles under automated longitudinal control mode as well as reject the leading vehicle acceleration and velocity. The control system is designed to be hierarchical[12], as shown in Figure 2. It is assumed that the leading vehicle is independent of the following vehicle, travels on its own, and there is no

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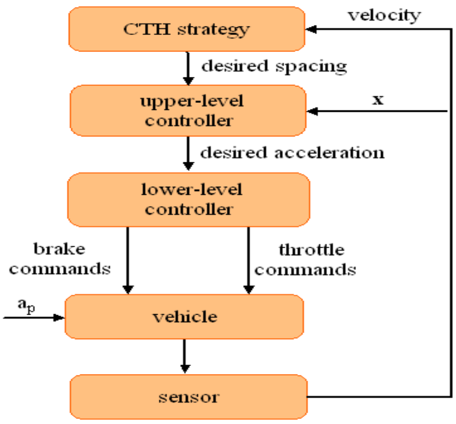

Figure 1 Two-level structure for longitudinal control system

communication between the two vehicles.

The desired spacing of the two vehicles can be obtained from spacing strategy, there exist two main spacing strategies: fixed spacing strategy and time-varying spacing strategy. Time headway based spacing appears as the most promising strategy[13]. In this paper, a simple constant time headway (CTH) strategy is adopted for the sake of simplicity. The longitudinal motion of the leading and following vehicle are measured by the upper level controller in order to calculate the desired acceleration sequence under some constraints for each vehicle, then selects the first element of the optimal solutions as the control law at current time. Since the desired acceleration is not a true control input, the lower level control is required to determine either a throttle or brake input in order to track the desired acceleration. The information of the following vehicle is fed back to the upper level controller to establish a feedback closed loop system. Then repeating the process until the ultimate following objectives is achieved.

3 The Longitudinal Dynamics Model of Inter-vehicle

The traditional method of modeling always takes the spacing and relative velocity between the two vehicles as state variables to get the second-order state equation, without considering the acceleration and velocity of the leading vehicle. However, it can't be ignored indeed due to the reduction of the model precision as well as the limit of the control algorithm design.

For a platoon of vehicles driving on one-lane road, shown as Figure 2. In order to avoid the collision, a car will strive to keep a safe distance when the traffic is heavy. The driver in the following car will take actions correspondingly according to the information of the leading car. The longitudinal dynamics model of inter-vehicle can be described by the following equation:

$$v(k+1) = v(k) + a_f(k)T_s \tag{1a}$$

$$\Delta x(k+1) = \Delta x(k) + v_{ref}(k)T_s + \frac{1}{2}(a_l(k) - a_f(k))T_s^2 \tag{1b}$$

where $a_l(k)$ and $a_f(k)$ stand for the acceleration of the leading vehicle and following vehicle respectively. $\Delta x(k)$ and $v_{ref}(k)$ stand for the spacing and relative velocity between the two vehicles, they can be described as: $\Delta x(k) = x_l(k) - x_f(k)$, $v_{ref}(k) = v_l(k) - v(k)$, where $x_l(k)$ and $x_f(k)$ denote the longitudinal position of the leading vehicle and following vehicle respectively; $v_l(k)$ and $v(k)$ denote the longitudinal speed of the leading vehicle and following vehicle respectively, while T_s is the sampling time of the system. Then choose state vector $X = [\Delta x(k), v(k)]^T$, input vector $u = [a_f]$, output vector $Y = [\Delta x(k)]$, and $w = [a_l, v_l]^T$ represents the acceleration and velocity of the leading vehicle which acts as an unknown disturbance input in this case, then the state-space model is given by:

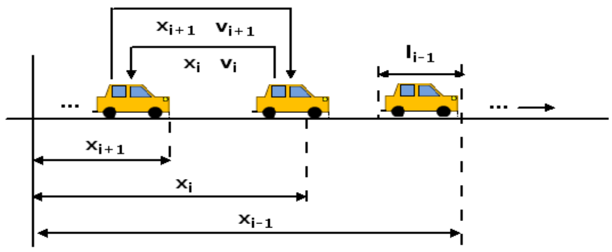

Figure 2 Vehicle platoon

$$X(k+1) = AX(k) + Bu(k) + Gw(k) \tag{2a}$$

$$Y(k+1) = CX(k) + Du(k) + Hw(k) \tag{2b}$$

where,

$$A = \begin{bmatrix} 1 & -T_s \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -\frac{1}{2}T_s^2 \\ T_s \end{bmatrix}, G = \begin{bmatrix} \frac{1}{2}T_s^2 & T_s \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & -T_s \end{bmatrix}, D = \begin{bmatrix} -\frac{1}{2}T_s^2 \end{bmatrix}, H = \begin{bmatrix} \frac{1}{2}T_s^2 & T_s \end{bmatrix}.$$

In this section, we choose the spacing and the longitudinal speed of the following vehicle as state variables, and take the acceleration and velocity of the leading vehicle as disturbance to ensure the precision of the model. This model describes the longitudinal dynamics of inter-vehicle more reliable compared with the traditional method.

4 Model Predictive Control

4.1 Ancillary Control Law

For the disturbance is unmeasurable and unpredictable, a novel MPC scheme is presented for linear stochastic systems with probabilistic constraints in [14]. The control signal is specified in terms of both nominal control action and an ancillary control law. We have also been able to find more similar details in [15], for linear systems subject to stochastic noise and probabilistic constraints on the state and control variables. The proposed method is characterized by a computational burden similar to the one that required by stabilizing MPC methods for deterministic systems, by the possibility to consider unbounded noises, and by ensured recursive feasibility and convergence.

Here we will explain how to get the ancillary control law. Let $w(k) = 0$, then define a nominal system:

$$\bar{X}(k+1) = A\bar{X}(k) + B\bar{u}(k) \tag{3a}$$

$$\bar{Y}(k+1) = C\bar{X}(k) + D\bar{u}(k) \tag{3b}$$

Denote $\alpha(k) = X(k) - \bar{X}(k)$ as the error between the actual system (2a) and the nominal system (3a). Let $u(k) =$

$\bar{u}(k) + K(X(k) - \bar{X}(k))$. Combine equation (2a) and (3a), the dynamics of the error system can be represented as:

$$\begin{aligned}\alpha(k+1) &= A\alpha(k) + B(u(k) - \bar{u}(k)) + Gw(k) \\ &= (A+BK)\alpha(k) + Gw(k)\end{aligned} \tag{4}$$

The ancillary control law forces the trajectories of the error system (4) to the origin, and the trajectory of system (2) is forced to the nominal and predicted trajectory, then achieves to disturbance minimization.

In simulations, we choose $Ts = 0.02s$, then from equation (2) we know that the pair (A, B) is stabilizable, for the matrix $[A - \lambda I, B]$ has full-row for all $Re\lambda \geq 0$. Then we just need to find the K satisfies that all the eigenvalues of $A + BK$ are located inside the unit circle.

Next, we will discuss how to solve the $\bar{u}(k)$.

4.2 Nominal Control Law

The nominal dynamics (3) rather than real dynamics (2) are used to predict the system behaviors, no stochastic disturbances are present.

According to the performance requirements, performance index that penalizes the tracking errors and vehicle acceleration desired to be small, so the objective function can be described as

$$\min_{U(k)}[J(Y(k), U(k), m, p)] \tag{5}$$

Satisfies the nominal dynamics of the system (3).

Because the ability of vehicle is restricted in practice, the following constraints must be satisfied.

$$Constraints1 : v_{min} \leq v(k) \leq v_{max}$$

$$Constraints2 : u_{min} \leq \bar{u}(k) \leq u_{max}$$

In addition, in order to avoid the collision, the car will strive to keep a safe distance.

$$Constraints3 : x_l(k) - x(k) \geq d_c$$

where d_c denotes the minimum safety distance, v_{min} and v_{max} stand for the minimum and maximum speed respectively. Finally, the optimization problem of longitudinal control can be described as follows:

Problem 1:

$$\min_{U(k)}[J(Y(k), U(k), m, p)] \tag{6}$$

satisfies the nominal dynamics of the system (3) and time-domain constraints:

$$u_{min} \leq \bar{u}(k+i|k) \leq u_{max} \tag{7a}$$

$$x_l(k+i|k) - x(k+i|k) \geq d_c \tag{7b}$$

$$v_{min} \leq v(k+i|k) \leq v_{max} \tag{7c}$$

Where,

$$\begin{aligned}&J(Y(k), U(k), m, p) \\ &= \sum_{i=1}^{m} \parallel \Gamma_{u,i}\bar{u}(k+i-1|k) \parallel^2 + \\ &\sum_{i=1}^{p} ||\Gamma_{y,i}(Y_c(k+i|k) - r(k+i)) \parallel^2 \\ &= ||\Gamma_y(Y_p(k+1|k) - R(k+1))||^2 + ||\Gamma_u U(k)||^2\end{aligned}$$

Matrices Γ_y and Γ_u are weighting factors, which are shown as

$$\begin{aligned}\Gamma_y &= diag(\Gamma_{y,1}, \Gamma_{y,2}, \cdots, \Gamma_{y,p}), \\ \Gamma_u &= diag(\Gamma_{u,1}, \Gamma_{u,2}, \cdots, \Gamma_{u,m}).\end{aligned}$$

$Y_p(k+1|k)$ is the prediction of the output, $R(k+1)$ is the reference values, $U(k)$ is the nominal control input sequences. Next, we will discuss these parts in detail.

According to the principles of model predictive control, at time k, the coming vehicle dynamics are predicted on the basis of model (3). Here, m is defined as the control horizon, p is defined as the prediction horizon, and $m \leq p$. Thinking that the whole state vector could be measured instantaneously. For the vehicle system, the input $\bar{u}(k)$ has no impact on $\bar{Y}(k+i|k)(i > 0)$, therefore, at time k, the prediction function of output is defined as follows:

$$Y_p(k+1|k) \stackrel{def}{=} \begin{bmatrix} \bar{Y}(k+1|k) \\ \bar{Y}(k+2|k) \\ \vdots \\ \bar{Y}(k+p|k) \end{bmatrix}_{p\times 1} \tag{8}$$

As well, define the optimal control input sequence $U(k)$ at time k:

$$U(k) \stackrel{def}{=} \begin{bmatrix} \bar{u}(k|k) \\ \bar{u}(k+1|k) \\ \vdots \\ \bar{u}(k+m-1|k) \end{bmatrix}_{m\times 1} \tag{9}$$

According to the basic principles of model predictive control and related theory, we can predict the future status of the system from the instance $k+1$ to $k+p$:

$$\bar{X}(k+1|k) = A\bar{X}(k) + B\bar{u}(k|k) \tag{10a}$$

$$\vdots$$

$$\begin{aligned}\bar{X}(k+m|k) = &A^m\bar{X}(k) + A^{m-1}B\bar{u}(k|k) + \cdots \\ &+ B\bar{u}(k+m-1|k)\end{aligned} \tag{10b}$$

$$\vdots$$

$$\begin{aligned}\bar{X}(k+p|k) = &A^p\bar{X}(k) + A^{p-1}B\bar{u}(k|k) + \cdots \\ &+ \sum_{i=1}^{p-m+1} A^{i-1}B\bar{u}(k+m-1|k)\end{aligned} \tag{10c}$$

Then combine (3b) with equation (10), the predictions of the controlled output $\bar{Y}(k+i|k)$ are obtained, which can be written as:

$$\bar{Y}(k+1|k) = C\bar{X}(k) + D\bar{u}(k|k) \tag{11a}$$

$$\vdots$$

$$\begin{aligned}\bar{Y}(k+m|k) = &CA^m\bar{X}(k) + CA^{m-1}B\bar{u}(k|k) + \cdots \\ &+ CB\bar{u}(k+m-1|k)\end{aligned} \tag{11b}$$

$$\vdots$$

$$\begin{aligned}\bar{Y}(k+p|k) = &CA^p\bar{X}(k) + CA^{p-1}B\bar{u}(k|k) + \cdots \\ &+ \sum_{i=1}^{p-m+1} CA^{i-1}B\bar{u}(k+m-1|k)\end{aligned} \tag{11c}$$

Then equation (11) can be described as:

$$Y_p(k+1|k) = S_x\bar{X}(k) + S_uU(k) \tag{12}$$

where,

$$S_x = \begin{bmatrix} CA & CA^2 & \cdots & CA^p \end{bmatrix}^T_{p\times 2},$$

$$S_u = \begin{bmatrix} CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{m-1}B & CA^{m-2}B & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{p-1}B & CA^{p-2}B & \cdots & \cdots & D \end{bmatrix}_{p\times(p+1)}.$$

Based on the CTH strategy, desired inter-vehicle spacing satisfies the following relations approximately:

$$r(k+i) = t_h v(k+i) + \Delta x_0 \tag{13}$$

where, Δx_0 denotes the minimum safety distance, including a vehicle length and a fixed distance value, $v(k+i)$ is the 2th system state, we can obtain the predictive output by equation (1a).

Similarly as before, define the predict output sequence $V_p(k+1|k)$:

$$V_p(k+1|k) \overset{def}{=} \begin{bmatrix} v(k+1|k) \\ v(k+2|k) \\ \vdots \\ v(k+p|k) \end{bmatrix}_{p\times 1} \tag{14}$$

Omitting the derivation, it is given by:

$$V_p(k+1|k) = V_x\bar{X}(k) + V_uU(k) \tag{15}$$

where,

$$V_x = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix}_{p\times 2},$$

$$V_u = \begin{bmatrix} T_s & 0 & 0 & 0 & \cdots & 0 \\ T_s & T_s & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ T_s & T_s & \cdots & \cdots & \cdots & T_s \end{bmatrix}_{p\times(p+1)}.$$

Define the reference output sequence as follows:

$$R(k+1) \overset{def}{=} \begin{bmatrix} r(k+1) \\ r(k+2) \\ \vdots \\ r(k+p) \end{bmatrix}_{p\times 1} \tag{16}$$

Then, it can be finally written as follows:

$$R(k+1) = t_h \times V_p(k+1|k) + R_0 \tag{17}$$

where,

$$R_0 \overset{def}{=} \begin{bmatrix} \Delta x_0 & \Delta x_0 & \cdots & \Delta x_0 \end{bmatrix}^T_{p\times 1}$$

We can not obtain the analytical solutions of this optimization problem due to the existence of the constraints, the optimization problem (6) subject to inequality constraints (7) can be formulated as a Quadratic Programming (QP) problem

$$\min_z z^THz - g^Tz, \tag{18a}$$

$$s.t.\ C_uz \geq b, \tag{18b}$$

where, $z = U(k)$ is the independent variable of this optimization problems. Substitute the predictive equation into the objective function, then, obtaining:

$$\begin{aligned} J &= ||\Gamma_y(E_p(k+1|k) + (S_u - t_hV_u)U(k)||^2 + ||\Gamma_uU(k)||^2 \\ &= U(k)^T((S_u - t_hV_u)^T\Gamma_y^T\Gamma_y(S_u - t_hV_u) + \Gamma_u^T\Gamma_u)U(k) \\ &+ 2E_p(k+1|k)^T\Gamma_y^T\Gamma_y(S_u - t_hV_u)U(k) \\ &+ E_p(k+1|k)^T\Gamma_y^T\Gamma_yE_p(k+1|k) \end{aligned} \tag{19}$$

where,

$$E_p(k+1|k) = S_x\bar{X}(k) - t_h \times V_x\bar{X}(k) - R_0$$

For $E_p(k+1|k)^T\Gamma_y^T\Gamma_yE_p(k+1|k)$ is independent to the variable. The cost function (19) can be rewritten as:

$$\tilde{J} = U(k)^THU(k) - G(k+1|k)^TU(k) \tag{20}$$

where,

$$H = (S_u - t_hV_u)^T\Gamma_y^T\Gamma_y(S_u - t_hV_u) + \Gamma_u^T\Gamma_u,$$

$$G(k+1|k)^T = 2(S_u - t_hV_u)^T\Gamma_y^T\Gamma_yE_p(k+1|k).$$

Next, we transform the constraints into $C_uz \geq b$.

★ *Transform control input constraint*

Constraint (7a) can be rewritten as $C_uz \geq b$

$$\begin{bmatrix} -I_{m\times m} \\ I_{m\times m} \end{bmatrix} U(k) \geq \begin{bmatrix} -u_{max} \\ \vdots \\ -u_{max} \\ u_{min} \\ \vdots \\ u_{min} \end{bmatrix} \tag{21}$$

★ *Transform output constraint*

Substituting the prediction equation (12) into (7b), then the output constraint can be rewritten as

$$S_uU(k) \geq D_c - S_x\bar{X}(k)$$

where,$D_c = [d_c\ \ldots\ d_c]^T_{p\times 1}$.

★ *Transform state constraint*

Combining equations (15), then the state constraint (7c) can be rewritten as

$$\begin{bmatrix} V_u \\ -V_u \end{bmatrix} U(k) \geq \begin{bmatrix} V_{min} - V_x\bar{X}(k) \\ V_x\bar{X}(k) - V_{max} \end{bmatrix} \tag{22}$$

where,

$$V_{min} = [v_{min}\ \cdots\ v_{min}]^T,$$

$$V_{max} = [v_{max} \ \cdots \ v_{max}]^T.$$

Based on the above analysis, the optimization problem Problem 1 can be finally described as Problem 2:

Problem 2:

$$\min_{U(k)} U(k)^T H U(k) - G(k+1|k)^T U(k), \tag{23a}$$

$$s.t.\ C_u U(k) \geq b(k+1|k), \tag{23b}$$

where,

$$H = (S_u - t_h V_u)^T \Gamma_y^T \Gamma_y (S_u - t_h V_u) + \Gamma_u^T \Gamma_u,$$

$$G(k+1|k)^T = 2(S_u - t_h V_u)^T \Gamma_y^T \Gamma_y E_p(k+1|k),$$

$$C_u = [-I_{m\times m}, I_{m\times m}, S_u, V_u, -V_u]^T,$$

$$b(k+1|k) = \begin{bmatrix} -u_{max} \\ \vdots \\ -u_{max} \\ u_{min} \\ \vdots \\ u_{min} \\ D_c - S_x \bar{X}(k) \\ V_{min} - V_x \bar{X}(k) \\ V_x \bar{X}(k) - V_{max} \end{bmatrix}_{(2m+3p)\times 1}.$$

It is clear that $H \geq 0$ and hence the optimal solution of the optimization problem exists.

By solving Problem 2, we can get the control sequence $U(k)$, only the first step of the optimal control sequence is applied to the controlled vehicle, then $\bar{u}(k) = [1 \ 0 \ \cdots \ 0]U(k)$, the control sequence $U^*(k)$ is described as follows:

$$U^*(k) = \bar{u}(k) + K(X(k) - \bar{X}(k)) \tag{24}$$

This is a process of rolling optimization.

5 Simulation

This paper implements some simulations by using co-simulation of Matlab/Simulink and veDYNA, a high fidelity vehicle simulator. In simulations, a vehicle model of limousine(Light) is utilized to show the effectiveness of the proposed method. Table 1 shows the main parameters used in simulation.

Table 1: The main parameters used in the simulation

Symbol	Description	Value
T_s	sampling time	0.02 (s)
l_0	vehicle length	4.3159 (m)
d_c	inter-vehicle constant distance	7 (m)
t_h	headway	6 (s)
u_{min}	the following vehicle min acceleration	-5 (m/s^2)
u_{max}	the following vehicle max acceleration	5 (m/s^2)
v_{min}	the following vehicle min velocity	0 (m/s)
v_{max}	the following vehicle max velocity	40 (m/s)
K_p	K_p in the lower level controller	12
K_i	K_i in the lower level controller	2
K_d	K_d in the lower level controller	0.01

In order to verify the effectiveness of the proposed method, several simulations are made.

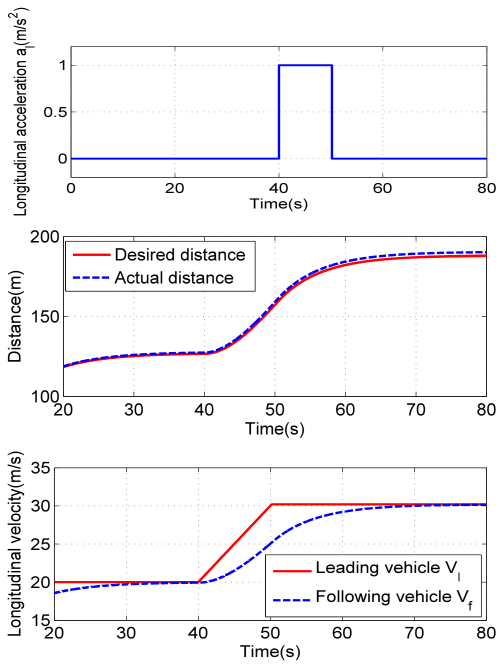

Figure 3 Experimental results with ancillary control law

A. Experimental validation of the proposed method

At first, we consider a simple set to verify the validity of the controller. Supposing that there is an initial offset of $100m$ between the two vehicles, and the leading vehicle keeps straight with the speed of $20m/s$, then there is an acceleration of $1m/s^2$ between $40s$ and $50s$. The acceleration of the leading vehicle and the detailed information are shown in Figure 3. In the above figure we omit the data before $20s$, for the state of following vehicle is not stable. According to Figure 3 it can be seen that facing of the leading vehicle's rapid change in the speed, the host car adopts a smoother tracking mode. In spite of the speed deviation, the host car can be able to track the desired distance perfectly. It is consistent with our expectation, for the purpose is to track the desired spacing between the two vehicles as well as restrain the change of the vehicle ahead.

B. Variation in road conditions

In this section, we set varied road friction to verify the tracking performance of the proposed control algorithm. Assuming that the variation in road conditions has no effect on the performance of the leading vehicle. In each case, other settings are the same including the parameter K. The path and other information can be seen in Fig 4. It is worth pointing out that, when the friction coefficient changes between $[0.4, 1]$, the host car can tracking the leading vehicle accurately. When it comes to the ice-covered road ($\mu = 0.2$), the controller becomes invalid.

C. Comparative experiments of the two controllers

To illustrate the ancillary control law can inhibit the acceleration and velocity of the leading vehicle, a sine change of speed in the leading vehicle is used in simulation. All the cases are implemented on dry asphalt pavement ($\mu = 1$). The vehicle states for different cases (with ancillary control law,without ancillary control law) are shown in Figure 5. We can notice that the host vehicle with ancillary control law can

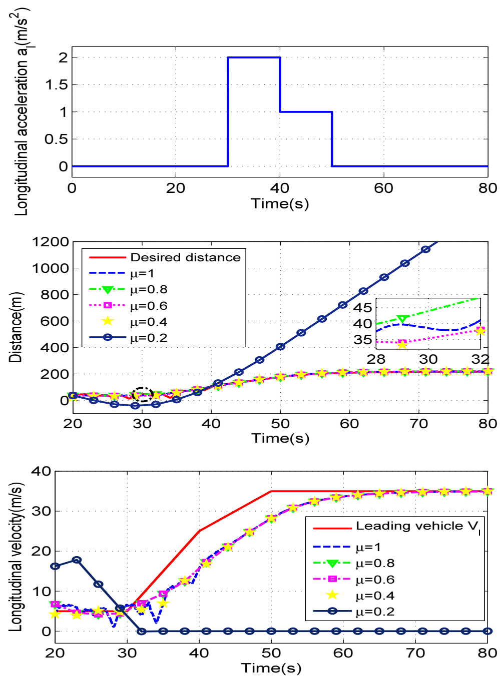

Figure 4 Comparative results of variation in road conditions

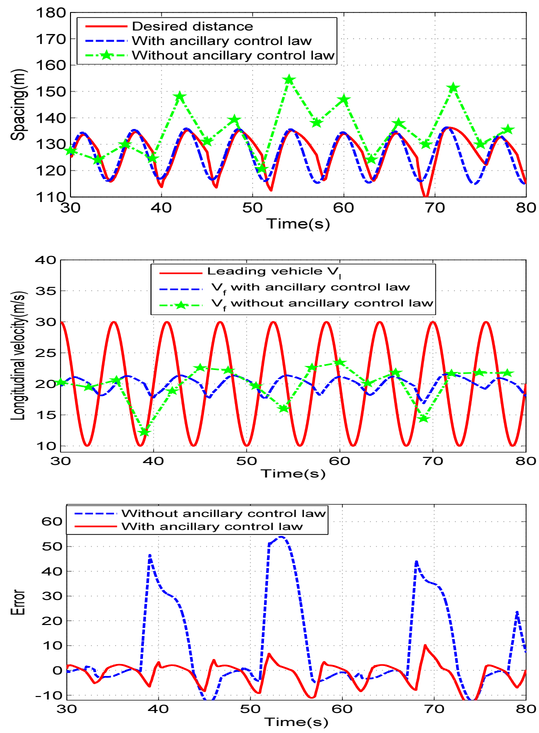

Figure 5 Comparative results of the two controllers

track the desired spacing with smaller error and longitudinal velocity fluctuations. All the vehicle states and responses are smoother than the other controller. Thus, such a conclusion can be obtained: the proposed method can track the desired spacing more accurately compared with the traditional control methods, and it can increase the driving comfort by reducing the vehicle longitudinal speed fluctuations.

6 Conclusion

This paper established a longitudinal dynamics model of inter-vehicle, which took the acceleration and velocity of the leading vehicle into consideration. For rejecting unknown disturbance, an ancillary control law was introduced under the framework of MPC. Simulation tests showed that the vehicle behaviour agreed with the proposed approach. The host vehicle for different cases (with ancillary control law and without ancillary control law) both could follow the leading vehicle, while ancillary control law helped the vehicle driving more smoothly by reducing the vehicle longitudinal acceleration fluctuations in the same situation.

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